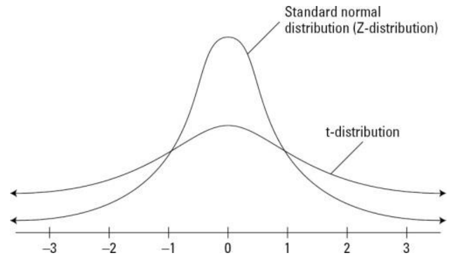
Introduction to the T-test

October 29, 2014

How is it different than a Z-test?



(1) Z-test is a statistical hypothesis test that follows a normal distribution, while T-test follows a Student's T-distribution.

(2) A T-test is appropriate when you are handling small samples ($n < 30$) while a Z-test is appropriate when you are handling moderate to large samples ($n > 30$).

-Note: as sample size increases, the t-distribution approaches the normal curve

(3) In research, T-tests are more commonly used than Z-tests.

(4) T-tests are best to use when you do not have the population values for the standard deviation and must estimate the standard deviation based on the sample data.

The General Steps of the T-test

→ The formula is nearly identical to that used for the Z-test. However, the standard error of the mean (aka, the S.D. of Sample Mean) is calculated differently

Step 1: Determine your hypotheses

Step 2: $SEM = \frac{S}{\sqrt{N}}$

Step 3: Determine your degrees of freedom and alpha level

Step 4: Use the Student's T-Distribution Table to find the T-critical Value
-find in Appendix E.6 on Page 590 in Howell text
-write down T-critical value as this will determine the region of rejection

Step 5: $t = \frac{\bar{x} - \mu}{SEM}$

Step 6: Compare your t-value and a t-critical values to determine whether you reject or retain your null hypothesis

Where do we get these components of these equations?

(1) $\bar{x}$ refers to our data -- our data provide the mean of our sample
(2) μ refers to our expectations under the null hypothesis (H_0), that is the mean of the population when the null hypothesis is true
(3) *SEM* is the standard error for the population mean. We do not know it and must estimate it from our sample standard deviation, and the equation corrects it based on our sample size.

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Example: T-test

The Boston College Admissions website states that the population mean performance for undergraduate students on some arbitrary exam is 20.3. We want to test to see if BC psychology majors (one sample of the entire BC population, N=5) perform statistically different on this arbitrary exam. You should run a two-tailed, non-directional test at $\alpha = .05$

Other Given Information:

$\bar{x} = 28$

Sample Scores: 29, 25, 24, 34, 28

Step 1: State hypotheses

$H_0 : \mu = 20.3$

$H_1 : \mu \neq 20.3$

Step 2: Calculate Standard Error of the Mean (SEM)

Calculate Sum of Squares

→ $SS = \sum(x - \bar{x})^2$

→ $SS = (x - \bar{x})^2 + (x - \bar{x})^2 + (x - \bar{x})^2 + (x - \bar{x})^2 + (x - \bar{x})^2$

→ $SS = (29\text{-}28)^2 + (25\text{-}28)^2 + (24\text{-}28)^2 + (34\text{-}28)^2 + (28\text{-}28)^2 = 62$

Calculate the Unbiased Variance (S^2)

→ $S^2 = \frac{SS}{N-1}$

→ $S^2 = \frac{62}{4} = 15.5$

Turn Unbiased Variance (S^2) into Standard Deviation of Sample Mean (S)

→ $S = \sqrt{S^2}$

→ $S = \sqrt{15.5} = 3.94$

Calculate the Standard Error of the Mean

→ $SEM = \frac{S}{\sqrt{N}}$

→ $SEM = \frac{3.94}{\sqrt{5}} = 1.76$ ⇦ This is called the Error Term

Step 3: Determine the Degrees of Freedom (d.f.) and Alpha value (α)

→ *d.f. = N -1 = 4*

→ $\alpha = .05$

Step 4: Use the Student's T-Distribution Table to find the T-critical Value

→Find in Appendix E.6 on Page 590 in Howell text

→ Look up *d.f.* in the left column to determine the appropriate row

→ Then determine the appropriate column to use based on alpha value (α)

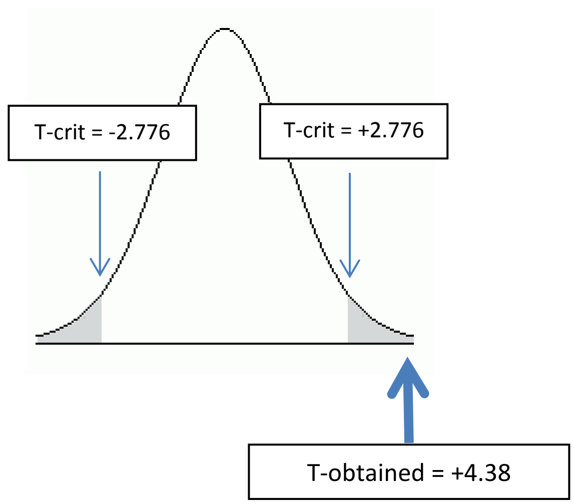
T-critical value = +/- 2.776

Step 5: Calculate the T-value for the sample to compare to T-critical

$$t - obtained = \frac{\bar{x} - \mu}{SEM}$$

$$t - obtained = \frac{28 - 20.3}{1.76} = +4.38$$

Step 6: Compare T-value to T-critical value. Does T-value fall in rejection region? Draw!



Note: *Rejection region is shaded in gray. If t-obtained falls in rejection region, then you always reject the null hypothesis and accept the research/alternative hypothesis.*

Step 7: Make Conclusion

→ Here, we reject the null hypothesis (H_0) because our T-value falls within the rejection region. Note: this rejection region was determined based on the T-Distribution Table, and based on degrees of freedom (N-1) and alpha ($\alpha = .05$). Therefore, we conclude that the sample of BC psychology students is *significantly* different (and in this case, better/higher) than the BC population on that arbitrary test score.